Inflation and the Joint Bond-FX Spanning Puzzle

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- "Spanning puzzle" in the bond asset pricing literature
  (e.g. Duffee 11, Joslin et al. 14)

Spanning condition:

- Affine term structure models imply macro variables do not predict returns after linearly controlling for current yield curve factors (level, slope, curvature . . .)
- Information in macro factors already included in yield curve factors

But: macro variables like inflation and measures of real activity often found to predict returns/ yield changes on top of yield factors

Summary of Results

Theoretical

- Show that linear spanning holds for bond returns also in non-linear models (e.g. habit model)
- Show that it holds also for FX excess returns

Empirical

- Find that inflation rate predicts not only US bond returns but also dollar returns
- Also holds conditional on yield curve factors

$\Rightarrow$ A *joint* spanning puzzle for FX and bond returns

Incomplete information about Fed's reaction function as likely explanation
$\Rightarrow$ Higher inflation predicts unexpected MP tightening (MP surprises)

Literature Review

"Spanning Puzzle" in the Bond Predictability Literature
e.g. Duffee (11), Joslin, Priebsch & Singleton (14), Cieslak & Povala (15), Bauer & Rudebusch (21)

Non-linear Macro-finance Models
e.g. Wachter (06), Rudebusch & Swanson (12)

Currency Predictability
e.g. Hassan & Mano (18), Lustig, Roussanov & Verdelhan (19), Dahlquist & Penasse (22)

Expectational Errors and Central Bank Reaction Function
e.g. Gourinchas & Tornell (04), Cieslak (17), Schmeling, Schrimpf & Steffensen (22), Bauer & Swanson (23)

Standard Linear Spanning (Duffee 11, Joslin et al. 14)

- Consider a linear model with m state variables
- m yield curve factors capture information in these variables
- No variable should predict bond returns / yield changes after controlling for these m factors
- Assumes a weak invertibility condition

Linear spanning in non-linear models (New result)

- Consider a non-linear model with m state variables, e.g. habit (Wachter 06) or long-run risk model (Bansal & Yaron 04)
- Show there is still approximate linear spanning but with more factors than state variables
- Approximation similar to a higher order local approximation
- Approximation accurate in standard models
- See here for the details

Linear spanning of FX returns (New result)

- Linear spanning for FX: using sufficiently many home and foreign yield curve factors should embed all necessary information relevant for future returns
- Macro variables such as inflation should not predict FX excess returns on top of these factors
- Requires additional but weak technical conditions

Data

- Focus on US bond returns and dollar returns against 5 countries (Canada, Germany, Sweden, Switzerland and UK)
- Easily available yield curve data from central banks
- Monthly data between 1973-2023 (FX data starts in 1983)
- Use BBI/WMI data for exchange rates
- Annual US CPI inflation rate from FRED (real time!)
- Monetary policy shocks based on GSS (18) and NS (18)
- Interest rate forecasts from Consensus Economics
- Taylor rule coefficients from Lombardi et al. (25)

Predicting Monthly Returns with Inflation

	Panel A: No YC Controls			
	(1)	(2)	(3)	(4)
	rx^{10Y}	rx^{FX}	Δy^{10Y}	Δs
π	-12.25***	15.78**	0.909*	15.67**
	(-2.73)	(2.00)	(1.73)	(2.03)
N	608	480	608	480
R^2 (in %)	1.22	1.07	0.58	1.07
	Panel B: With YC Controls			
	(1)	(2)	(3)	(4)
	rx^{10Y}	rx^{FX}	Δy^{10Y}	Δs
π	-14.46**	27.04***	1.682**	25.27***
	(-2.29)	(3.15)	(2.36)	(3.07)
N	608	480	608	480
R^2 (in %)	2.88	4.87	2.11	3.92

t statistics in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Predicting Monthly Returns with Inflation: Summary

- 1 pp higher inflation leads to 12bps lower bond and 16bps higher dollar return
- Effects mainly due to changes in yields and dollar FX rates
- Predictability survives controlling for yield curve factors $\Rightarrow$ spanning condition is violated
- How about mechanisms?

Predicting MP Shocks with Inflation

	Panel A: No YC Controls		
	(1)	(2)	(3)
	Target	Path	NS
π	0.0584	1.149***	0.306**
	(0.47)	(2.30)	(2.39)
N	231	231	231
R^2 (in %)	0.06	3.46	2.30
	Panel B: YC Controls		
	(1)	(2)	(3)
	Target	Path	NS
π	0.148	1.14**	0.358**
	(0.89)	(2.22)	(2.37)
N	231	231	231
R^2 (in %)	5.53	11.15	11.99

t statistics in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Predicting Yield Forecast Errors and Changes in Taylor Rule Coefficient on Inflation Using Inflation

	Panel A: Long rate forecast errors	
	(1)	(2)
	No YC Controls	YC Controls
π	13.51	12.74**
	(1.53)	(2.47)
N	395	395
R^2(%)	5.04	42.09

	Panel B: Changes in the Taylor coefficient on inflation	
	(1)	(2)
	No YC Controls	YC Controls
π	0.168	0.219**
	(1.01)	(2.09)
N	405	405
R^2(%)	2.49	18.73

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Rationalizing the results: a small model

- We do not take a strong stance on the specific model that explains the results
- However, we provide a stylized example of a model that could
- Assume risk neutral investors
- Fed sets the short rate according to $r_t = \phi \bar{\pi}_t + v_t$
- $\bar{\pi}_t$ is long run inflation and v_t is a random shock
- Assume $\bar{\pi}_t$ follows AR(1) but is unobserved by the agents

Rationalizing the results: a small model

- Agents short rate expectations follow a sticky expectations process

$$\mathbb{E}_t^S[r_{t+1}] = k\lambda\mathbb{E}_{t-1}^S[r_t] + (1-k)\lambda r_t$$

- This emerges as a solution to a filtration problem with unknown $\bar{\pi}_t$
- But it could also represent a simple behavioral rule.
- Forecast under FIRE: $\lambda\phi\bar{\pi}_t$
- High inflation predicts short rate increases
- The short rate increases lead to low bond returns and currency appreciation
- Inflation is also unspanned by yield curve factors

Conclusion

- Conceptual contribution: extend linear spanning to non-linear models and to FX rates
- Find inflation rates predict not only US bond returns but also dollar appreciation

$\Rightarrow$ Bond spanning puzzle is a *joint* bond-FX spanning puzzle ...

- High inflation also predicts unexpected MP tightening
- Rationalize the results with a simple model, where agents have incomplete information about Fed's reaction function.

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Appendix: References

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Schmeling, M., Schrimpf, A., and Steffensen, S. (2022). Monetary policy expectation errors. *Journal of Financial Economics*, 146(3):841–858.

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Appendix: Linear Spanning in Non-linear Models

- Two countries. Home state variables $x_t \in \mathbb{R}^{m \times 1}$ and foreign state variables $x_t^* \in \mathbb{R}^{m^* \times 1}$
- Yield of an n maturity home bond $y_t^n \equiv g(x_t)$, g generally non-linear
- Excess bond return $rx_{t,t+1}^n = -(n-1)y_{t+1}^{n-1} + ny_t^n - y_t^1$
- Bond risk premium: $\mathbb{E}_t[rx_{t,t+1}^n] \equiv \Pi_n(x_t)$
- (Log) exchange rate s_t (higher $s_t \rightarrow$ dollar appreciation)
- Dollar excess return $s_{t+1} - f_t$, f_t is forward rate
- FX risk premium $\mathbb{E}_t[s_{t+1} - f_t] \equiv f(x_t^*) - f(x_t)$

- We approximate $y_t^n = g(x_t)$ by $y_t^n \approx A_n + B_n YCF_t$
- YCF_t is a vector of yield curve factors.
- To capture non-linearities can need more factors than state variables
- Approximation similar to a higher order local approximation
- Similarly $\mathbb{E}_t[rx_{t,t+1}^n] \approx C_n + D_n YCF_t$
- and $\mathbb{E}_t[s_{t+1} - f_t] \approx F_n + H_n YCF_t + H_n^* YCF_t^*$
- Expected Bond and FX returns only depend on yield curve factors!
- Nothing else should predict these returns once linearly controlling for them.

Appendix: Linear Spanning in Non-linear Models

- Show numerically that approximations accurate in standard models (e.g. habit)
- Results for bonds only require a weak invertibility condition
- For currencies they also require a separability condition (holds in standard models)
- Relaxing separability, need to also control for interactions between home and foreign factors
- Argue empirically that this does not change the results