Discussion of

"Inflation and the Joint Bond–FX Spanning Puzzle"

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How Do FX Researchers Think About Interest Rates?

- Uncovered Interest Parity (UIP) $\mathrm{E}_t\left[s_{t+1} - s_t\right] = -(y_t^{*1} - y_t^1)$.
  - Convention: s_t is the log exchange rate in dollars per unit of foreign currency.
- Testable implication (Bilson 1981; Fama 1984; Tryon 1979), $\beta = 1$ in:

$$s_{t+1} - s_t = \alpha - \beta\left(y_t^{*1} - y_t^1\right) + \varepsilon_{t+1}.$$

- Adding $y_t^{*1} - y_t^1$ to both sides gives the currency excess return regression (Fama, 1984):

$$rx_{t+1}^{FX} = \alpha + (1-\beta)\left(y_t^{*1} - y_t^1\right) + \varepsilon_{t+1}.$$

  - Empirically $\beta < 0$: UIP puzzle

How Do Term-Structure Researchers Think About FX?

Inflation and the Joint Bond-FX Spanning Puzzle

- Andreas and Markus propose to study FX through the term-structure lens.
- Two dimensions in the paper:
  - Spanning: link between bond and currency risk premia.
  - Inflation disconnect: inflation predicts both FX and bond returns.
- I first discuss the spanning dimension.
- Intuitively: FX similar to long-term bonds. both are ‘interest-rate sensitive’ assets heavily exposed to news about future short-term bonds (Greenwood, Hanson, Stein, and Sunderam, 2022)
- This motivates a detour to the level of the exchange rate.

Present-Value Decomposition of the Exchange Rate

Campbell and Clarida (1987)

- One-period currency excess return:

$$rx_{t+1}^{FX} = s_{t+1} - s_t + y_t^1 - y_t^{*1}$$

- Rearranging:

$$s_t = s_{t+1} + (y_t^{*1} - y_t^1) - rx_{t+1}^{FX}$$

- Iterating forward and taking expectations:

$$s_t = \mathrm{E}_t[s_{t+h}] + \sum_{j=0}^{h-1} \mathrm{E}_t\left[y_{t+j}^{*1} - y_{t+j}^1\right] - \sum_{j=1}^{h} \mathrm{E}_t\left[rx_{t+j}^{FX}\right]$$

- Letting $h \to \infty$:

$$s_t = \omega_t + \sum_{j=0}^{\infty} \mathrm{E}_t\left[y_{t+j}^{*1} - y_{t+j}^1\right] - \sum_{j=1}^{\infty} \mathrm{E}_t\left[rx_{t+j}^{FX}\right]$$

where $\omega_t = \lim_{h\to\infty} \mathrm{E}_t[s_{t+h}]$ reflects long-run PPP and price index differentials.

Digression: FIRE vs Subjective Beliefs

Implicit FIRE assumption in Campbell–Clarida

$$s_t = \omega_t + \sum_{j=0}^{\infty} \mathrm{E}_t \left[y^{*1}_{t+j} - y^{1}_{t+j} \right] - \sum_{j=1}^{\infty} \mathrm{E}_t \left[rx^{FX}_{t+j} \right]$$

One could also explore similar relation using subjective beliefs:

$$s_t = \omega_t + \sum_{j=0}^{\infty} \mathrm{E}^S_t \left[y^{*1}_{t+j} - y^{1}_{t+j} \right] - \sum_{j=1}^{\infty} \mathrm{E}^S_t \left[rx^{FX}_{t+j} \right]$$

Putting some Structure

Dahlquist–Pénasse (2022)

- Take UIP regression and ignore constant terms

$$rx_{t+1}^{FX} = (1-\beta)\left(y_t^{*1} - y_t^1\right) + \varepsilon_{t+1}.$$

- Interest rate differential follows an AR(1):

$$y_{t+1}^{*1} - y_{t+1}^1 = \rho_i\left(y_t^{*1} - y_t^1\right) + \varepsilon_{t+1}^i.$$

- Combining with the Campbell–Clarida decomposition gives:

$$s_t - \omega_t = \beta\,\frac{y_t^{*1} - y_t^1 - \mu_i}{1-\rho_i}.$$

- In this one-factor world, the exchange rate level co-moves almost mechanically with the short-rate differential ⇒ counterfactual in the data.

Augmenting UIP with a Persistent Risk Premium

Dahlquist–Pénasse (2022)

- DP augment the UIP-type equation with a missing risk premium:

$$rx^{FX}_{t+1} = \alpha + (1-\beta)\left(y^{*1}_t - y^1_t\right) + \eta_t + \varepsilon_{t+1},$$

  where $\eta_{t+1} = \rho_\eta \, \eta_t + \varepsilon^{\eta}_{t+1}$.

- Combining with Campbell–Clarida gives the representation:

$$s_t - \omega_t = \beta \, \frac{y^{*1}_t - y^1_t - \mu_i}{1-\rho_i} - \frac{\eta_t}{1-\rho_\eta}, \qquad \omega_t = \lim_{h\to\infty} \mathrm{E}_t[s_{t+h}].$$

- Interpretation:
  - Empirically, $\eta_t \approx 100\%$ in the (real) exchange rate variance.
  - Parallel with the stock market: requires a persistent, time-varying risk premium.
  - But η_t has little structural content: may include convenience yields or higher-order yield factors.

Connecting Yield Curves and the Dollar

- Markus & Andreas propose the spanning condition:

$$\mathrm{E}_t[rx_{t+1}^{FX}] \;\approx\; F' T_t + F^{*'} T_t^* .$$

- For exposition, assume that both the short-rate differential and the FX risk premium load on global yield-curve factors Γ_t:

$$y_t^{*1} - y_t^1 = a_\Gamma' \Gamma_t, \qquad rx_{t+1}^{FX} = \lambda' \Gamma_t + \varepsilon_{t+1}.$$

  - International CAPM logic: FX $\mathrm{RP}_t = \sum_i \mathrm{price}_{i,t} \times \mathrm{quantity}_{i,t}$

- Assuming stationary VAR for global factors, the present value of future interest differentials and risk premia satisfy:

$$\sum_{j=0}^{\infty} \mathrm{E}_t\left[y_{t+j}^{*1} - y_{t+j}^1\right] = \kappa' \Gamma_t, \qquad \sum_{j=1}^{\infty} \mathrm{E}_t\left[rx_{t+j}^{FX}\right] = \Lambda' \Gamma_t.$$

with coefficients κ, Λ determined by the VAR.

From Global Yield Factors to the Dollar Level

Campbell–Clarida: $$s_t - \omega_t = \sum_{j=0}^{\infty} \mathrm{E}_t \left[y_{t+j}^{*1} - y_{t+j}^{1} \right] - \sum_{j=1}^{\infty} \mathrm{E}_t \left[rx_{t+j}^{FX} \right]$$

Use $$\sum_{j=0}^{\infty} \mathrm{E}_t \left[y_{t+j}^{*1} - y_{t+j}^{1} \right] = \kappa' \Gamma_t, \qquad \sum_{j=1}^{\infty} \mathrm{E}_t \left[rx_{t+j}^{FX} \right] = \Lambda' \Gamma_t.$$

$$\Rightarrow \qquad s_t - \omega_t = (\kappa - \Lambda)' \Gamma_t$$

- Potential solution to the missing risk premium puzzle: the dollar co-moves with global yield-curve factors Γ_t.
- Problem: Andreas and Markus find yield factors have little forecasting power for currency returns (see also Chernov and Creal, 2023).
- Multi-factor analog of the low correlation between interest rate differentials and exchange rates.

Inflation

- Core empirical finding: US inflation predicts both bond and currency excess returns, even controlling for term-structure factors.

$$r^{bond}_{t+1} = a_b + b_b\,\pi^{US}_{t-1} + F_b'\,T_t + F_b^{*'}\,T_t^* + \varepsilon^b_{t+1},$$

$$rx^{FX}_{t+1} = a_{fx} + b_{fx}\,\pi^{US}_{t-1} + F_{fx}'\,T_t + F_{fx}^{*'}\,T_t^* + \varepsilon^{fx}_{t+1}.$$

- Paper argues this predictability is not a risk-premium channel:
  - Survey evidence: high inflation followed by unexpected monetary tightening.
  - Tightening $\Rightarrow$ dollar appreciation and low long-term bond returns.

Main Spanning Test

- A tentative replication for currency returrns with two samples:
  - Andreas–Markus (5 countries, 1983–2023)
  - Dahlquist–Pénasse (9 countries, 1976–2020)
- Note: Andreas & Markus sample starts later due to missing early yield-curve data.
- I ignore the term-structure controls, but consider the interest-rate differential and real exchange rate (q_t) as controls following Dahlquist–Pénasse:
- Regression:

$$rx^{FX}_{t+1} = a + b_\pi \, \pi^{US}_{t-1} + b_i \, (y^{*1}_t - y^1_t) + b_q \, q_t + \varepsilon_{t+1}.$$

A tentative replication (cont'd)

Relation seems weaker in the earlier period:

	Andreas–Markus sample			Dahlquist–Pénasse sample		
π_{t-1}	-0.19^{**}	-0.22^{*}	-0.20	-0.08^{*}	-0.05	-0.02
	(-2.43)	(-1.95)	(-1.61)	(-1.83)	(-1.20)	(-0.52)
d_t		1.67^{*}	2.02^{**}		1.81^{**}	2.14^{***}
		(1.75)	(2.07)		(2.55)	(2.98)
q_t			-0.01			-0.01
			(-1.24)			(-1.44)
R^2	0.02	0.02	0.03	0.01	0.02	0.03
N	480	438	438	519	519	519

Wrapping Up

- Thought-provoking paper bridging two macro–finance literatures.
- I had lots of fun preparing this discussion.
- Main suggestions:
  - How much of the exchange rate variance is explained by yield curve factors?
  - Explore a longer sample (e.g., using global factors).
  - Allow for a potentially time-varying relation: more forecast errors in the recent period?

Appendix: Connecting Dollar Returns and Global Factors (I)

- Assume FX premia and interest differentials load on a small set of global yield-curve factors Γ_t:

$$y_t^{*1} - y_t^1 = a_\Gamma' \Gamma_t, \qquad rx_{t+1}^{FX} = \lambda' \Gamma_t + \varepsilon_{t+1}.$$

- Assume the global factors follow a stationary VAR:

$$\Gamma_{t+1} = A\Gamma_t + u_{t+1}, \qquad \mathrm{E}_t[\Gamma_{t+j}] = A^j \Gamma_t.$$

- All eigenvalues of A lie strictly inside the unit circle $\Rightarrow$ the geometric sum $(I - A)^{-1}$ exists.

Appendix: Connecting Dollar Returns and Global Factors (II)

- Expected future short-rate differentials:

$$\mathrm{E}_t[y^{*1}_{t+j} - y^{1}_{t+j}] = a'_\Gamma \mathrm{E}_t[\Gamma_{t+j}] = a'_\Gamma A^j \Gamma_t.$$

- Present value:

$$\sum_{j=0}^{\infty} \mathrm{E}_t[y^{*1}_{t+j} - y^{1}_{t+j}] = a'_\Gamma (I - A)^{-1} \Gamma_t \equiv \kappa' \Gamma_t.$$

- Expected future FX risk premia:

$$\mathrm{E}_t[rx^{FX}_{t+j}] = \lambda' \mathrm{E}_t[\Gamma_{t+j-1}] = \lambda' A^{j-1} \Gamma_t, \qquad j \geq 1.$$

- Present value:

$$\sum_{j=1}^{\infty} \mathrm{E}_t[rx^{FX}_{t+j}] = \lambda' (I - A)^{-1} \Gamma_t \equiv \Lambda' \Gamma_t.$$

- Both PV objects are linear in the same global state Γ_t. If (T_t, T^*_t) span Γ_t, the spanning condition follows.